# Background information:

## Assumption: Black Scholes Frame

We use assumptions from the Black-Scholes Equation that the continuously compounded annual return rate on a stock for the period from to follows a Normal distribution . This implies that , where is the expected return and is the volatility of the stock. Note that the mean is expressed as , but .

To further complicate things, the stock return is assumed to be a Brownian Motion. The only implication we will use is the property that the return on any day is independent from the return on any other day.

## Bayesian Inference

Suppose we would like to estimate a parameter of a specified distribution with density function . Ordinarily we would gather a sample of observations and estimate the parameter with such that the likelihood function is maximized at .

However, with Bayesian Inference, we have a prior belief of the distribution of . The prior belief can come from historical data or physical theory, or it can be purely subjective, and the distribution which we believe that follows is called the prior distribution with density . Then, with the same observation, we can calculate the “posterior density” of given our observations:

From there we can estimate with which maximizes the posterior density.

# Model

## Stock return

The return on stock from time to time , , follows a normal distribution . Taking period to be (1 month), we have Note that “iid” means “independently and identically distributed”, and is the return on the -thmonth.

Taking and , we have

Where

## Formulate prior distribution from analyst report

On analyst report, they usually specify the target price at one year later. We can take that number as their belief of the expected stock price at . Which implies that

However, to turn the point belief into a density, it is reasonable to assume that the return follow a normal distribution centered at . As for variance, before we obtain any more information, we would assume that we are 60% sure that we won’t lose anything, and that gives , from which we can calculate . Then we have

and

The analyst report itself did not provide any information regarding the volatility of the stock, so we need to estimate the volatility using other methods.

## Regression of stock volatility on VIX

In order to determine the current stock volatility, firstly we note that it is inappropriate to use the estimated volatility from the past few years, since the volatility was very low during those years in the market as a whole. So instead, we would like to use VIX as a guidance to estimate the current volatility of the stock.

We use linear regression:

Then we will plug in the current VIX to obtain the stock volatility .

## Posterior density of return rate

Using the likelihood function and prior density for , we can formulate the posterior density for as:

Where

## Estimate of

Maximize posterior density with respect to to get the estimate

Then we have estimated the parameters for the stock price

## Estimate of covariance between stocks

We use sample variance from the historical prices from each pair of stocks to estimate the covariance between them.

## Capital Allocation Line (CAL)

We model CAL with one-year return random variable (subscript now refers to an individual stock). From above calculations, we let and . From here we can determine the optimal portfolio using the CAL method.